# NUMERICAL STUDY OF HEAT SOURCE/SINK EFFECTS ON DISSIPATIVE MAGNETIC NANOFLUID FLOW FROM A NON-LINEAR INCLINED STRETCHING AND SHIRINKING SHEET

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# DEDICATION

To my beloved Father, Mother and Siblings

And my supervisor and friends that always help in finishing the writing

Special friend from Kedah that always give support from beginning.

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## ABSTRACT

This research numerically investigate radiative MHD mixed convection boundary layer flow of nanofluid over a nonlinear inclined stretching and shrinking sheet in the presence of heat source/sink and vicious dissipation. The governing coupled nonlinear momentum and thermal boundary layer equation are transform into a system of ordinary differential equations via similarity transformation with appropriate boundary condition. The dimensionless parameters that used in this study are magnetic field parameter, volume fraction parameter, power-law parameter, Richardson number, suction and injection parameter, Eckert number, heat source and heat sink parameter. A detailed study of the influence of these parameters on velocity and temperature distribution is conducted. The skin friction coefficient and rate of heat transfer values with selected parameters is presented.

ABSTRAK

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# LIST OF SYMBOLS AND ABBREVIATION

**Nomenclature**

| | |
|---|---|
| $(u, v)$ | - Velocity components along $(x, y)$ axes |
| $u_w$ | - Stretching/Shrinking sheet velocity |
| $\psi$ | - Stream function |
| $\eta$ | - Similarity variable |
| $f$ | - Dimensionless velocity |
| $g$ | - Acceleration due to gravity |
| $m$ | - Nonlinear parameter |
| $b$ | - Constant |
| $c$ | - Stretching, >0 and shrinking, <0 parameter |
| $M$ | - Dimensionless magnetic field parameter |
| $S$ | - Suction/Injection parameter |
| $Q$ | - Heat Source/Sink parameter |
| $Nr$ | - Thermal radiation parameter |
| $\mathrm{Re}_x^2$ | - Reynolds number |
| $Ri$ | - Richardson number |
| $Ec$ | - Eckert number |
| $Nu_x$ | - Nusselt number |
| $\mathrm{Pr}$ | - Prandtl number |
| $\mathrm{Pr}_{eff}$ | - Effective Prandtl number |
| $C_f$ | - Skin friction coefficient |
| $Gr$ | - Thermal Grashof number |
| $B$ | - External uniform magnetic field |
| $B_\circ$ | - Magnetic field strength |
| $q_r$ | - Thermal radiative heat flux |

| | |
|---|---|
| $Ks$ | - Thermal conductivity of the solid |
| $K_f$ | - Thermal conductivity of fluid |
| $K_{nf}$ | - Thermal conductivity of nanofluid |
| $T$ | - Local temperature of the fluid |
| $T_w$ | - Temperature of the fluid at the wall |
| $T_\infty$ | - Free stream temperature |
| $k*$ | - Mean absorption coefficient |

**Greek Symbols**

| | |
|---|---|
| $\alpha$ | - Thermal diffusivity coefficient |
| $\alpha_f$ | - Thermal diffusivity of the fluid |
| $\alpha_s$ | - Thermal diffusivity of the solid |
| $\alpha_{nf}$ | - Thermal diffusivity of the nanofluid |
| $\beta$ | - Thermal expansion coefficient |
| $\beta_f$ | - Coefficient of thermal expansion of the fluid |
| $\beta_s$ | - Coefficient of thermal expansion of the solid |
| $\rho_f$ | - Density of the fluid friction |
| $\rho_s$ | - Density of the solid friction |
| $\rho_{nf}$ | - Density of the nanofluid |
| $v$ | - Kinematic viscosity |
| $v_f$ | - Kinematic viscosity of the fluid |
| $\mu$ | - Dynamic viscosity |
| $\mu_f$ | - Dynamic viscosity of the fluid |
| $\mu_{nf}$ | - Dynamic viscosity of the nanofluid |
| $\sigma$ | - Electrical conductivity |
| $\sigma_s$ | - Electrical conductivity of the solid |
| $\sigma_f$ | - Electrical conductivity of the fluid |

| | |
|---|---|
| $\sigma_{nf}$ | - Electrical conductivity of the nanofluid |
| $\sigma^{*}$ | - Stefan-Blotzmann constant |
| $c_p$ | - Specific heat at constant pressure |
| $(\rho c_p)_{nf}$ | - Heat capacitance of the nanofluid |
| $(\rho c_p)_f$ | - Heat capacitance of the fluid |
| $(\rho c_p)_s$ | - Heat capacitance of the solid |
| $\varphi$ | - Nanoparticle volume fraction parameter |
| $\theta$ | - Non-dimensional temperature |

**Subscripts**

| | |
|---|---|
| $f$ | - Fluid Phase |
| $s$ | - Solid Phase |
| $Nf$ | - Nanofluid |
| $w$ | - Condition at the wall |
| $\infty$ | - Condition at freestream |

# LIST OF APPENDICES

# CHAPTER 1

# INTRODUCTION

## 1.1 Research Background

In the recent years the field of science and technology, nanotechnology has become more popular because of its specific application to the arenas of electronic, fuel cells, space, fuels, better air quality, batteries, solar cells, medicine, cleaner water, chemical sensors and sporting goods. In the understanding of all these features, there is an important field acknowledged as the nanofluid, which is fundamentally a homogeneous mixture of the base fluid and nanoparticles.

Nanofluid are engineered colloids made of a base fluid and nanoparticles (1-100nm). Common base fluid include water, organic liquid (e.g. ethylene, tri-ethylene-glycols, etc.), oils and lubricants, bio-fluids, polymeric solutions and other common liquids. Materials commonly used as nanoparticles include chemical stable metals (e.g. gold and copper), oxide ceramics (e.g. $Al_2O_3, CuO$), metal carbides (e.g. SiC), metal nitrides (e.g. AIN,SiN), carbon in various forms (e.g. diamond, graphite, carbon nanotube, fullerene) and functionalized nanoparticles (Cheng, 2009). Nanofluid commonly contain up to 5% volume friction of nanoparticles which is effective enhancement of heat transfer.

The convectional heat transfer fluids like water, oil, kerosene and ethylene glycol have poor heat transfer capabilities due to their low thermal conductivity. To improve the thermal conductivity of these fluids nano/micro-sized materials are suspended in liquids (Pakravan & Yaghoubi, 2011). Existence of the suspension is to increase the heat transfers coefficient. The present of the particles can increase the thermal conductivity of the fluid and also increase heat transfer performance which caused by the present of nanometal is higher than the base fluids (Das, 2015). There are several advantages of nanofluids which is stable, have sufficient viscosity and better wetting which is spreading and dispersion properties on solid surface even for modest nanoparticles concentration. Therefore, due to the nanofluid thermal enhancement, performance, applications and benefits in several important arenas, the nanofluid has contributed significantly well in the field of microfluidics, manufacturing, microelectronics, advanced nuclear systems, polymer technology, transportation, and medical (Yogeswara, P. R. & Raju, Dr.G.S.S., 2017).

The study of magnetic field effects has important application in engineering and physics. In several engineering process, such as materials manufactured by extrusion process and heat treated materials traveling between a feed roll and wind-up roll on convey belts possess the characteristics of a moving continuous surface (Cortell, 2014). In polymer engineering, the hydromagnetic technique are being used. In the view of this application, studies of magnetohydrodynamic (MHD) flow of Newtonian fluid initially at rest, over a stretching sheet at different uniform temperature has been done by Chakrabarti and Gupta (1979).

### 1.1.1 Advantages of Nanofluids

Particle size is the major physical parameter in nanofluids, since it can be used to attune the nanofluid thermal properties as well as the suspension stability of nanoparticles. Hence, nanofluid able to flow freely through mini and micro channels with the dispersion of nanoparticles. The nano suspension show high thermal conductivity which is mainly due to enhanced convection between the nanoparticles and base liquid surfaces. Another pontential benefit is that the nanoparticles have lower dimension so that the dispersed nanoparticles seems to be like a base fluid molecule in suspension.

### 1.1.2 Magnetohydrodynamic (MHD)

The term "Magnetohydrodynamic" or MHD explains magnetic property of the electrical conducting fluids. The reason of using MHD is to induce the current in the fluid flow to conduct electricity. The presence of MHD layer change the polarize fluid into magnetic field fluid. The study of nanofluids in the presence of magnetic fields has important applications in many area for example physics and engineering.

## 1.2 Problem Statement

To investigate radiative magnetohydrodynamic mixed convection baoundary layer flow of nanofluid from a nonlinear inclined stretching and shrinking sheet in the presence of heat source/sink effects and viscous dissipation with variable stream condition will be focused in this study. The main focus is to find the relationship between heat source/sink effects in the presence of water-based Cu and $Al_2O_3$ with the influences of magnetic field and the effects of physical parameters on temperature and velocity distribution. Types of nanoparticle shape used in this study is laminar, $e = 16.1576$. There are several factors need to be considered in this study. Firstly, changing the partial differential equations (PDE) to Ordinary differential equation (ODE). Besides that, chose the optimum values for each parameter in order to obtain optimum heat and mass transfer of nanofluids is taken into consideration as well.

## 1.3 Objectives

The objectives of this study is:

i. To transform governing nonlinear momentum and thermal boundary layer equation into an ordinary differential equation by using similarity transformation.
ii. To study the heat source/sink effect onto viscous dissipation of different water based nanofluids from an inclined non-linear shrinking sheet.
iii. To analyses the effects of various governing physical parameter on velocity and temperature distribution for stretching and shrinking sheet

## 1.4 Scope of Study

In order to achieve all objectives, it is important to outline the scope of study to avoid any diverted. There are there scope have been defined and listed as follow:

i. This study will focus on laminar, steady heat transfer and 2-dimensional of the incompressible of MHD dissipative flow of an electrically-conducting nanofluid from an inclined stretching or shrinking sheet, oriented at an angle $\alpha\left(0^{\circ} \leq \alpha \leq 90^{\circ}\right)$ to the vertical.

ii. The mathematical model is solved using Range Kutta shooting method and Maple 18 programming to get the graphical result.

iii. Governing nonlinear momentum and thermal boundary layer equation is transform into an ordinary differential equation by using similarity transformation.

## 1.5 Significance of Study

The aim of this study, to understand the effects of various parameter on magnetic field and physical parameter on temperature and velocity for stretching and shrinking sheet. The solution will provide the engineers and researchers with a graphical graph and data concerning skin friction and rate of heat transfer for nanofluid. From this data, it will help better understanding on behavior of nanofluid.

## 1.6 Thesis Outline

Chapter 1: Establish the purpose of the research, problem statement, the scope of the research and method to use.

Chapter 2: Relevant literature review about the heat transfer and thermal conductivity of nanofluid, nanoparticles, magnetohydrodynamic, stretching and shrinking sheet.

Chapter 3: Present the method that will used to solve the non-linear ordinary differential equations by using shooting method.

Chapter 4: Present the result and discussion that have been obtained from the method that have been used.

Chapter 5: Present the conclusion of the objectives that have been achieved. Some recommendation for the future work also mentioned in this chapter.

Table 1.7.1: The Gantt Chart for Master Project 1 and gives the outlines and durations to complete the task. Master Project 1 must be covered from Chapter 1 until Chapter 3.

Table 1.7.2: The Gantt Chart for Master Project 2 and gives the outlines and durations to complete the task. Master Project 1 must be covered whole project and result. The important part of this chart is result and discussion.

## 1.7 Gantt Chart

### 1.7.1 Master Project 1

Table 1.7.1: Gantt chart for Master Project 1.

| Project Activity | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| Confirmation of thesis title | | | | | | | | | | | | | |
| Master project Briefing | | | | | | | | | | | | | |
| Chapter 1<br>(Introduction)<br>1. Research background<br>2. Problem statement<br>3. Objective<br>4. Importance of study | | | | | | | | | | | | | |
| Mid semester break | | | | | | | | | | | | | |
| Chapter 2<br>(Literature Review) | | | | | | | | | | | | | |
| Chapter 3<br>(Methodology)<br>1. Basic equation<br>2. PDE<br>3. ODE | | | | | | | | | | | | | |
| Submission of report | | | | | | | | | | | | | |
| Presentation of master project 1 | | | | | | | | | | | | | |

### 1.7.2 Master Project 2

Table 1.7.2: Gantt chart for Master Project 2.

| Project Activity | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| Starting new semester | | | | | | | | | | | | | |
| Updating previous chapter (Chapter 1 until Chapter 3) | | | | | | | | | | | | | |
| Chapter 4 ( Result and discussion) | | | | | | | | | | | | | |
| Mid semester break | | | | | | | | | | | | | |
| Chapter 5 (Conclusion) 1. Conclusion 2. Recommendation | | | | | | | | | | | | | |
| Submission report to supervisor | | | | | | | | | | | | | |
| Submission of report to panel | | | | | | | | | | | | | |
| Presentation of master project 2 | | | | | | | | | | | | | |

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 Heat Transfer of Nanofluids

Choi (1995) was the first who introduced a new type of fluid called nanofluid while doing research on new coolants and cooling technologies. Eastman, Choi, Li, Thompson & Lee (1997) have noticed in an experiment that the thermal conductivity of the base fluid (water) has increased up to 60% when CuO nanoparticles of volume fraction 5% are added to the base fluid. This is because of increasing surface area of the base fluid due to the suspension of nanoparticles. Eastman, Choi, Yu & Thompson (2001) have also showed that the thermal conductivity has increased 40% when copper nanoparticles of volume fraction less than 1% are added to the ethylene glycol or oil.

### 2.1.1 Thermal Conductivity of Nanofluids

A wide range of experimental and theoretical studies were conducted in the literature to model thermal conductivity of nanofluids. The existing results were generally based on the definition of the effective thermal conductivity of a two-component mixture. The Maxwell (1881) model was one of the first models proposed for solid–liquid mixture with relatively large particles. It was based on the solution of heat conduction equation through a stationary random suspension of spheres.

Thermal conductivity signifies the inherent ability of heat transfer and it is very important property for all thermal applications involving fluids. Heat conduction is depends upon thermal conductivity. Furthermore, Nusselt number, an important parameter in convective heat transfer is directed related to thermal conductivity of fluids (Purna, Sayantan & Santosh, 2013). Eastman, Lee, Choi, & Li (1999) reported that an

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### Chapter 1


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